

TWO-DIMENSIONAL HIGHER-DERIVATIVE SUPERGRAVITY AND A NEW MECHANISM FOR SUPERSYMMETRY BREAKING

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ABSTRACT. We discuss the general form of quadratic $(1, 1)$ supergravity in two dimensions, and show that this theory is equivalent to two scalar supermultiplets coupled to non-trivial supergravity. It is demonstrated that the theory possesses stable vacua with vanishing cosmological constant which spontaneously break supersymmetry.

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1. INTRODUCTION

In two recent papers [1, 2], we discussed theories of higher-derivative bosonic gravitation in four dimensions. Such theories have higher-order equations of motion for the metric and describe, in addition to the helicity-two graviton, extra scalar and symmetric-tensor degrees of freedom. We presented a method for reducing such theories to a canonical second-order form by introducing the new degrees of freedom explicitly through a Legendre transformation, the exact analog of forming the Helmholtz Lagrangian to reduce a second-order theory to first-order. Using the second-order form, we explored the vacuum structure of these theories and showed, in particular, the existence of non-trivial vacua which have a non-negligible effect on low-energy physics. However, it turned out that all such non-trivial vacua must have non-vanishing cosmological constant and, hence, correspond to either deSitter or anti-deSitter spacetime with a radius generically of the order of the inverse Planck mass. It is little wonder that such non-trivial gravity vacua have played no role in particle physics to date.

In this paper, we continue to explore higher-derivative gravitational theories but with two modifications. First, we restrict our discussion to two dimensions and second, and most importantly, we introduce supersymmetry, analyzing the vacuum structure of higher-derivative $(1, 1)$ supergravity. We find that these theories continue to exhibit non-trivial vacua, with a richer structure, in fact, than in the bosonic case. Remarkably, we find that these non-trivial vacua can now have vanishing cosmological constant and, hence, correspond to flat spacetime. Exactly what this means, and how it occurs, will be explicitly discussed. The reason that the cosmological constant can now vanish, where it could not in bosonic gravity, can be

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directly traced to the fact that in higher-derivative supergravity the ostensibly auxiliary field in the gravity supermultiplet becomes a new, propagating degree of freedom. This phenomenon was first presented in the context of $D = 4$, $N = 1$ supergravity by Ferrara, Grisaru, and van Nieuwenhuizen [3]. This new degree of freedom, by extending the range of vacuum solutions, allows vacuum states with zero energy. This result opens the door to non-trivial supergravity vacua playing a role in particle physics. One is led to ask whether such vacua have any demonstrable physical effect. The answer is a resounding yes. We will show that generically these non-trivial, flat spacetime supergravitational vacua spontaneously break supersymmetry! It seems plausible to us that, if this result persists in four-dimensional, $N = 1$ supergravity, it represents a new approach to spontaneous supersymmetry breaking in phenomenological supergravity and, perhaps, superstring theories [4]. We have recently shown that this phenomenon does, indeed, exist in four dimensions. This work will be presented elsewhere [5].

This paper is organized as follows. In Section 2, we discuss higher-derivative bosonic gravity in two dimensions, introducing the method of Legendre transformations and reducing these theories to canonical second-order form. In Section 3, such theories are generalized to quadratic $(1, 1)$ supergravity. The structure of these theories is discussed and, using a supersymmetric generalization of the method of Legendre transformations, they also are reduced to a canonical second-order form. A similar method, within the context of $D = 4$, $N = 1$ supergravity using the compensator formalism, was given in [6]. The main results of this paper are to be found in Section 4. Here we restrict ourselves to a specific class of models and explore their vacuum structure. We demonstrate explicitly that they generically contain non-trivial vacua with vanishing cosmological constant and that these vacua spontaneously break supersymmetry. We close the section by showing how $(1, 1)$ supersymmetry allows the cosmological constant at non-trivial vacua to vanish. We present our conclusions and a few closing remarks in Section 5. Relevant details about $(1, 1)$ supergravity [7], as well as the notation we will use, are given in a brief Appendix.

2. GRAVITY IN TWO DIMENSIONS

Einstein gravity in two dimensions is trivial, since its action,

$$S = \int d^2x \sqrt{-g} R, \quad (2.1)$$

is an integral over a total divergence. This can be easily seen by using the fact that any two-dimensional space is conformally flat. Hence, we can always choose the conformal gauge in which the metric is given by

$$g_{mn} = e^\sigma \eta_{mn}. \quad (2.2)$$

A straightforward calculation then shows that the Lagrangian in the above action is just $\nabla^2 \sigma$.

How would one go about writing non-trivial gravity theories in two dimensions? One way is to include a scalar field, λ . A simple non-trivial action for the metric g_{mn} and the scalar field λ can be written as

$$S = \int d^2x \sqrt{-g} e^\lambda R. \quad (2.3)$$

Since the term $\sqrt{-g} e^\lambda R$ is not a total divergence, the equations of motion of both gravity and the scalar field λ are non-trivial. We can naturally generalize action (2.3) by adding an arbitrary potential term for λ ,

$$S = \int d^2x \sqrt{-g} [e^\lambda R - V(\lambda)]. \quad (2.4)$$

The equations of motion for λ and g_{mn} derived from (2.4) are

$$\begin{aligned} R &= e^{-\lambda} \frac{dV}{d\lambda}, \\ \nabla_m \nabla_n e^\lambda - g_{mn} \nabla^2 e^\lambda &= \frac{1}{2} g_{mn} V(\lambda). \end{aligned} \quad (2.5)$$

respectively. The physical content of this theory is most easily extracted by choosing the conformal gauge (2.2). In this gauge, the action becomes

$$S = \int d^2x [e^\lambda \nabla^2 \sigma - e^\sigma V(\lambda)]. \quad (2.6)$$

The fields σ and λ enter the above Lagrangian on, more or less, an equal footing. Expanding the integrand as a power series in λ and σ yields

$$S = \int d^2x [\nabla^2 \sigma + \lambda \nabla^2 \sigma - m^2 \lambda^2 + \dots] \quad (2.7)$$

where $m^2 = d^2V/d\lambda^2|_{\lambda=0}$. The first term is a total divergence, so can be dropped. The second and third terms are quadratic kinetic energy and mass terms respectively. All other terms are higher-order interactions that we will ignore for the time being. In order to diagonalize the quadratic kinetic energy term in the action, we make the field redefinitions

$$\phi_+ = \frac{\lambda + \sigma}{2}, \quad \phi_- = \frac{\lambda - \sigma}{2}. \quad (2.8)$$

Solving for λ and σ in terms of ϕ_+ and ϕ_- , and substituting back into the quadratic piece of the action (2.7), yields

$$S_Q = \int d^2x [-\nabla^m \phi_+ \nabla_m \phi_+ + \nabla^m \phi_- \nabla_m \phi_- - m^2 (\phi_+ + \phi_-)^2]. \quad (2.9)$$

The field ϕ_+ has a proper kinetic energy term and, hence, is a physically propagating degree of freedom. However, ϕ_- has a kinetic energy term with the opposite sign. It follows that it is a degree of freedom with ghost-like propagating behavior. The origin of this ghost-like behavior is clearly the off-diagonal quadratic coupling inherent in the $e^\lambda R$ term. In four dimensions one can remove the λ - R coupling by performing a conformal transformation of

the metric. However, in two dimensions this is no longer true. Consider the conformal transformation

$$\bar{g}_{mn} = \Omega^2 g_{mn}, \quad (2.10)$$

where Ω is an arbitrary conformal factor, to be chosen later. A little manipulation shows that the action (2.4) takes the form

$$S = \int d^2x \sqrt{-\bar{g}} [e^\lambda \bar{R} + 2e^\lambda \bar{\nabla}^2 \ln \Omega - \Omega^{-2} V(\lambda)]. \quad (2.11)$$

This conformally transformed action has the same λ - R cross term. We cannot get rid of this coupling, no matter how we choose the conformal factor. This is in accordance with the previous statement that action (2.4) describes two degrees of freedom. If we had succeeded in reducing the gravity part of the action to pure Einstein form, the gravity sector of the theory would be trivial and the action could describe only a single degree of freedom, that of the field λ . Be this as it may, one might still want to make a conformal transformation, in order to grow an explicit kinetic energy term for λ , and put the action in a more canonical form. Let us choose the conformal factor to be

$$\Omega = e^\lambda. \quad (2.12)$$

The conformally transformed action then takes the form,

$$S = \int d^2x \sqrt{-\bar{g}} [e^\lambda \bar{R} - 2e^\lambda (\bar{\nabla} \lambda)^2 - e^{-2\lambda} V(\lambda)]. \quad (2.13)$$

The net effect of the conformal transformation is to grow an explicit kinetic energy term for λ , with a sigma-model factor in front of it. The equations of motion for λ and \bar{g}_{mn} obtained by varying action (2.13) are

$$\begin{aligned} \bar{R} &= -2(\bar{\nabla} \lambda)^2 - 4\bar{\nabla}^2 \lambda - 2e^{-3\lambda} V(\lambda) + e^{-3\lambda} \frac{dV}{d\lambda}, \\ \bar{\nabla}_m \bar{\nabla}_n e^\lambda - \bar{g}_{mn} \bar{\nabla}^2 e^\lambda - e^\lambda \bar{g}_{mn} (\bar{\nabla} \lambda)^2 + 2e^\lambda \bar{\nabla}_m \lambda \bar{\nabla}_n \lambda &= \frac{1}{2} \bar{g}_{mn} e^{-2\lambda} V(\lambda). \end{aligned} \quad (2.14)$$

respectively. Obviously, if we work in the \bar{g}_{mn} frame with action (2.13), and then chose conformal gauge, we will get different formulae for the fields ϕ_+ and ϕ_- , but one of them will certainly be a ghost-like. This situation is unavoidable in such theories. It is important to note that the \bar{g}_{mn} equations of motion for constant field λ_0 immediately require that $V(\lambda_0) = 0$. The λ equation simply determines \bar{R} given λ_0 . Thus, the condition that constant λ_0 be a vacuum solution of the theory is given by $V(\lambda_0) = 0$ and the value of $dV/d\lambda|_{\lambda_0}$ need not be specified. It is only if we further demand that the spacetime has zero cosmological constant, that is $\bar{R} = 0$, that we must take $dV/d\lambda|_{\lambda_0} = 0$. This is exactly the reverse of the situation in all dimensions greater than two, where the vacuum condition is that $dV/d\lambda|_{\lambda_0} = 0$, whereas $V(\lambda_0) = 0$ implies vanishing cosmological constant.

The non-trivial gravity theory that we considered, defined by action (2.4), seems a priori to be an unmotivated and ad hoc choice. However, as we will now show, this is not the case. First, let us consider an apparently unrelated way of getting non-trivial gravity in two dimensions by introducing higher-derivative gravitational terms. In four dimensions there are, in addition to R , two more tensors that can be used in constructing the Lagrangian, namely the Ricci tensor and the Riemann tensor. In two dimensions, however, both the Ricci and Riemann tensors can be expressed in terms of R . Hence, we have only one tensor at our disposal for constructing Lagrangians. The simple choice of R itself as the Lagrangian was discussed above and is trivial. However, choosing an arbitrary function of R for the Lagrangian yields non-trivial gravitation, as we will now show. Consider the action

$$S = \int d^2x \sqrt{-g} f(R), \quad (2.15)$$

where f is an arbitrary real function of the scalar curvature R . The scalar curvature R is a function of the metric field g_{mn} , its first-order derivatives $\partial_\ell g_{mn}$, and its second-order derivatives $\partial_p \partial_\ell g_{mn}$. Hence, the equations of motion for the metric field of the above action (2.15) are expected to be fourth-order equations. Such theories are referred to as higher-derivative theories of gravitation. The importance of studying higher-derivative theories in two dimensions is not only that they provide, as we will show, a way of making gravity non-trivial, but that they also mimic many properties of higher-derivative theories of gravity in four dimensions, which arise in phenomenologically relevant theories such as supergravity models and string theory.

The equations of motion for the metric g_{mn} derived from the above action are

$$f' R_{mn} - \frac{1}{2} f g_{mn} + g_{mn} \nabla^2 f' - \nabla_m \nabla_n f' = 0, \quad (2.16)$$

which, for a generic choice of f , are fourth-order differential equations as expected. Using the fact that, in two dimensions, the Ricci tensor can be written as

$$R_{mn} = \frac{1}{2} g_{mn} R, \quad (2.17)$$

we can write the equations of motion in the form

$$\nabla_m \nabla_n f' - g_{mn} \nabla^2 f' = \frac{1}{2} g_{mn} (f' R - f). \quad (2.18)$$

Let us make the identification

$$e^\lambda = f'(R), \quad (2.19)$$

which can be inverted to give the scalar curvature R in terms of the field λ as

$$R = X(e^\lambda) \quad (2.20)$$

where X denotes the functional inverse of f' . Furthermore, define a potential energy for λ as

$$V(\lambda) = e^\lambda X(e^\lambda) - f(X(e^\lambda)). \quad (2.21)$$

With these identifications, equation (2.18) is in exactly the same form as the second equation in (2.5), with the potential energy specified in (2.21). Also note that the first equation in (2.5) is also satisfied as can easily be shown by differentiating the potential in (2.21). Therefore, the higher-derivative equation of motion (2.18) is equivalent to the two second-order equations of motion (2.5). Of course, if we conformally transform (2.18) and compare it with (2.14), we will be able to make the same identification, although we have to work a little harder.

This equivalence can be established on the level of the actions by the method of Legendre transformations. This elegant procedure is, in fact, much easier to apply. We start by introducing an auxiliary field X and the transformed action

$$S = \int d^2x \sqrt{-g} [f'(X)(R - X) + f(X)]. \quad (2.22)$$

The auxiliary field X has the equation of motion

$$f''(X)(R - X) = 0 \quad (2.23)$$

Provided that $f''(X) \neq 0$, this gives $X = R$, which when substituted into (2.22), gives back action (2.15). Now we can define a scalar field $\lambda = \ln f'(X)$, such that the action (2.22) represents a Legendre transform from the variable R to the variable e^λ . Writing the above action in terms of λ we find that

$$S = \int d^2x \sqrt{-g} [e^\lambda R - V(\lambda)], \quad (2.24)$$

where

$$V(\lambda) = e^\lambda X(e^\lambda) - f(X(e^\lambda)). \quad (2.25)$$

Comparing this result with action (2.4), we conclude that the generic higher-derivative gravitation theory described by action (2.15) is equivalent to the non-trivial gravity-plus-scalar theory discussed earlier. Of course, one can also perform a conformal transformation on the metric g_{mn} to put the theory in the canonical form (2.13), if one so desires.

As a concrete example of this formalism, let us consider the quadratic higher-derivative action

$$S = \int d^2x [R + \epsilon R^2]. \quad (2.26)$$

We introduce an auxiliary field X and write an equivalent action to (2.26) as

$$S = \int d^2x [(1 + 2\epsilon X)(R - X) + (X + \epsilon X^2)]. \quad (2.27)$$

The equation of motion of X is

$$X = R. \quad (2.28)$$

Substituting (2.28) into (2.27) gives the original higher-derivative action (2.26). This establishes the equivalence of the higher-derivative action (2.26) and the second-order action (2.27). Now define

$$e^\lambda = 1 + 2\epsilon X. \quad (2.29)$$

Using this definition, action (2.27) becomes

$$S = \int d^2x [e^\lambda R - V(\lambda)], \quad (2.30)$$

where $V(\lambda)$ is given by

$$V(\lambda) = \frac{1}{4\epsilon}(e^\lambda - 1)^2. \quad (2.31)$$

To conclude, by writing a higher-derivative theory of gravity, not only did we make gravity a non-trivial propagating degree of freedom, but we also introduced another propagating degree of freedom, the field λ . Classically the theory is completely equivalent to the gravity-plus-scalar theory described by the action (2.4). Furthermore, one of these degrees of freedom is ghost-like.

3. SUPERGRAVITY IN TWO DIMENSIONS

Supercharges are decomposable into left- and right-chiral species. In two dimensions, the supersymmetry algebra can have p left supercharges and q right supercharges. This is referred to as (p, q) supersymmetry. In this paper, we will be interested in $(1, 1)$ supersymmetry only, since this is the closest analog to the phenomenologically relevant $N = 1$ supersymmetry in four dimensions. The theory of $(1, 1)$ supergravity was studied by Howe [7], and we will use his results and notation. We present the relevant formulae, and set the notation, in the Appendix. Howe found that the supergravity multiplet consists of a graviton e_m^a , a gravitino χ_a^α and an auxiliary field A . All the geometrical quantities in $(1, 1)$ superspace, such as the curvature and the torsion, can be expressed in terms of these component fields. Two important superfields are the superdeterminant

$$E = e \left(1 + \frac{i}{2} \theta^\alpha \gamma^\alpha{}_\alpha{}^\beta \chi_{a\beta} + \bar{\theta} \theta \left[\frac{i}{4} A + \frac{1}{8} \epsilon^{ab} \chi_a^\alpha \gamma^\alpha{}_\alpha{}^\beta \chi_{b\beta} \right] \right), \quad (3.1)$$

and the real scalar superfield

$$S = A + i\theta^\alpha \Sigma_\alpha + \frac{i}{2} \bar{\theta} \theta C, \quad (3.2)$$

where

$$\begin{aligned} C &= -R - \frac{1}{2}\chi_a{}^\alpha\gamma^\alpha{}_\alpha{}^\beta\psi_\beta + \frac{i}{4}\epsilon^{ab}\chi_a{}^\alpha\gamma^5{}_\alpha{}^\beta\chi_{b\beta}A - \frac{1}{2}A^2, \\ \Sigma_\alpha &= -2\epsilon^{ab}\gamma^5{}_\alpha{}^\beta\mathcal{D}_a\chi_{b\beta} - \frac{1}{2}\gamma^a{}_\alpha{}^\beta\chi_{a\beta}A. \end{aligned} \quad (3.3)$$

If S vanishes, so does the curvature and the superspace is flat. Einstein supergravity in two dimensions is given by

$$S = 2i \int d^2x d^2\theta ES. \quad (3.4)$$

If we expand this action in components, we find that there is no contribution from the gravitino $\chi_a{}^\alpha$, while the bosonic part is just Einstein gravity (2.1). Therefore, minimal supergravity in two dimensions is also trivial, with no propagating degrees of freedom.

How would one go about writing non-trivial supergravity theories in two dimensions? We can try to supersymmetrize the non-trivial gravity theories of the previous section. The supersymmetrization of gravity coupled to a scalar field of the form (2.4) was discussed by various authors [8, 9]. In this paper, we are more interested in the supersymmetric analog of action (2.15); that is, in higher-derivative supergravitation.

A naive supersymmetrization would be to consider a Lagrangian that is a general function of the superfield S . However, since the spacetime curvature scalar R occurs as the highest component of S , any function $f(S)$, prior to the elimination of auxiliary fields, will contain only R rather than arbitrary powers of R . The equation of motion of the A field is algebraic, which means A is an auxiliary field. Eliminating A from the Lagrangian, we find, generically, that A is expressed in terms of fractional powers of R . Inserting the solution of the algebraic equation of motion of A into the Lagrangian does lead to higher powers of R . However, for all but the simplest functions $f(S)$, the equation of motion of A cannot be solved in closed form. For this reason, we will not consider such theories in this paper. A simpler supersymmetrization is to consider Lagrangians that contain higher powers of R prior to any elimination of auxiliary fields.

With this in mind, we would like to construct a superfield that has R as its lowest component. We can form such a superfield by taking the θ -derivative of S twice. Since we need a covariant object, we have to consider $\mathcal{D}^\alpha\mathcal{D}_\alpha S$. This superfield, along with S itself, can be used to construct superfields with arbitrary higher powers of the spacetime curvature R . However, it can be easily shown that any quadratic or higher power of $\mathcal{D}^\alpha\mathcal{D}_\alpha S$ leads to equations of motion that are fourth-order in the field A . A fourth-order equation of motion of a scalar field contains a ghost-like degree of freedom [10]. For this reason, we demand that the Lagrangian does not contain any powers of $\mathcal{D}^\alpha\mathcal{D}_\alpha S$ higher than unity. Are there any other covariant terms that can be used to construct a higher-derivative supergravity Lagrangian? Any terms with higher derivatives with respect to θ , for example $\mathcal{D}^\alpha\mathcal{D}_\alpha\mathcal{D}^\beta\mathcal{D}_\beta S$, or spacetime derivatives, such as $\nabla^m\nabla_m S$, lead to $\nabla^m\nabla_m R$ in the component field action. If we demand

that the supersymmetric theory be at most fourth-order, then terms like $\mathcal{D}^\alpha \mathcal{D}_\alpha \mathcal{D}^\beta \mathcal{D}_\beta S$ and $\nabla^m \nabla_m S$ are excluded.

According to the above discussion, the general fourth-order supergravity action is given by

$$S = 2i \int d^2x d^2\theta E[f(S) + ig(S)\mathcal{D}^\alpha S \mathcal{D}_\alpha S], \quad (3.5)$$

where f and g are two arbitrary real functions of the superfield S . Note that we have made an integration by parts in writing the Lagrangian in the above form. What is the dynamical content of action (3.5)? If we expand the above action in components, using the expressions introduced in the Appendix, and perform the θ integrals, we obtain a Lagrangian of the following form

$$\mathcal{L} = \mathcal{L}_{\text{Boson}} + \mathcal{L}_{\text{Fermion}} + \mathcal{L}_{\text{Boson-Fermion}}, \quad (3.6)$$

where

$$\mathcal{L}_{\text{Boson}} = e \left[-(f'(A) - 2g(A)A^2)R - 2g(A)R^2 + 2g(A)(\nabla A)^2 - \frac{1}{2}(f'(A)A^2 + g(A)A^4) \right]. \quad (3.7)$$

The $\mathcal{L}_{\text{Fermion}}$ and $\mathcal{L}_{\text{Boson-Fermion}}$ terms are rather complicated expressions and will not be needed here. The explicit appearance of R^2 confirms that Lagrangian (3.5) is indeed a supersymmetrization of R^2 gravity. Note that no higher-powers of R appear. Thus despite including two arbitrary functions f and g , we only have a supersymmetric extension of R^2 rather than of any arbitrary power of R . The $\mathcal{D}^\alpha S \mathcal{D}_\alpha S$ term not only has the effect of introducing R^2 directly in the Lagrangian, but also of growing a kinetic energy term for the scalar field A , as can be seen from (3.7). What does this mean? It means that the field A , is no longer auxiliary. It is now a propagating field. How many degrees of freedom are described by the bosonic part of the action given in (3.7)? As we have seen in the previous section, $R + R^2$ gravity describes two real degrees of freedom. Hence, (3.7) describes three real degrees of freedom, two coming from the higher derivative R^2 term, and the third being the once auxiliary field A .

Since the action (3.5) is supersymmetric, one should expect supersymmetric partners for these three degrees of freedom. There should be three fermions propagating along with these three bosonic fields. Indeed, a direct computation of the fermionic part of the Lagrangian shows that the equation of motion of the gravitino $\chi_a{}^\alpha$ is third-order. A single fermionic degree of freedom is described by a first-order differential equation. Only one initial condition is required to solve the Cauchy problem of such a field. A higher-order differential equation implies the existence of more degrees of freedom. In particular, a third-order differential equation describes three fermionic degrees of freedom, since three initial conditions

are required to solve the Cauchy problem. These are the three fermionic degrees of freedom associated with the three bosonic degrees of freedom.

To better understand the dynamical content of (3.5), we would like to transform it into a second-order theory, in much the same way as we transformed the bosonic $R + R^2$ theory in (2.26) into a second-order theory in the previous section. Hence, we are tempted to introduce a superfield in much the same way we introduced the scalar field λ in the previous section. But, as discussed before, $R + R^2$ bosonic gravity describes only two degrees of freedom, one of which is the graviton. Therefore a single real field λ is all that is required to describe the extra degree of freedom and to transform the theory to second-order form. Here, as we have shown, there are three degrees of freedom, one of which is the graviton. Thus we need two superfields to describe the extra degrees of freedom and to transform the theory into a second-order form. We will denote these real superfield by Φ and Λ . Apart from this subtlety, there is not much difference between the method of reduction as applied to bosonic theories or supersymmetric theories. Action (3.5) is equivalent to the second-order action

$$\begin{aligned} S &= 2i \int d^2x d^2\theta E [f(\Phi) + ig(\Phi) \mathcal{D}^\alpha \Phi \mathcal{D}_\alpha \Phi + e^\Lambda (S - \Phi)] \\ &= 2i \int d^2x d^2\theta E [e^\Lambda S + ig(\Phi) \mathcal{D}^\alpha \Phi \mathcal{D}_\alpha \Phi + f(\Phi) - e^\Lambda \Phi]. \end{aligned} \quad (3.8)$$

The superfield equations of motion of Λ and Φ are given by

$$\begin{aligned} \Phi &= S, \\ e^\Lambda &= f'(S) - ig'(S) \mathcal{D}^\alpha S \mathcal{D}_\alpha S - 2ig(S) \mathcal{D}^\alpha \mathcal{D}_\alpha S, \end{aligned} \quad (3.9)$$

respectively. Here Λ acts as a Lagrange multiplier, with the effect of setting Φ equal to S . Substituting $\Phi = S$ into (3.8) gives us back the original action (3.5). Action (3.8) is the supersymmetric extension of the bosonic action (2.4). The superfield Φ is a propagating superfield with an explicit kinetic energy term. The superfield Λ is also a propagating superfield, due to its coupling with the superfield S . It is clear that quadratic supergravity is equivalent to non-trivial supergravity (due to the e^Λ factor in front of S) coupled to two new scalar superfield degrees of freedom.

What exactly are the new propagating degrees of freedom in terms of the original variables e_m^a , χ_a^α , and A ? Gravity itself is propagating, so the graviton e_m^a and the gravitino χ_a^α are propagating degrees of freedom. Besides this, the equations of motions of Φ and Λ give the new degrees of freedom in terms of the original variables. Consider Φ first. It follows from (3.9) and (A.10) that

$$\begin{aligned} \Phi| &= A, \\ \Phi|_\theta &= -2\epsilon^{ab} \gamma^5_\alpha{}^\beta \mathcal{D}_a \chi_{b\beta} - \frac{1}{2} \gamma^a_\alpha{}^\beta \chi_{a\beta} A. \end{aligned} \quad (3.10)$$

Therefore, A is a propagating degree of freedom, as we have already seen. Its fermionic superpartner is a complicated function of the first derivative of the gravitino and A itself. Secondly, consider Λ . The lowest component of Λ is set equal to the natural logarithm of the lowest component of the second line in (3.9), which is a particular, but involved, function of both A and R . The supersymmetric partner of this field, namely, the fermionic field in Λ , is even more involved as a function of the original fields. However, at the linearized level, the components of Λ can be evaluated. They are given by

$$\begin{aligned}\Lambda| &= \ln f'(0) + \frac{f''(0)}{f'(0)}A + \frac{2g(0)}{f'(0)}R, \\ \Lambda|_\theta &= -\frac{2f''(0)}{f'(0)}\epsilon^{ab}\gamma^5{}_\alpha{}^\beta D_a\chi_{b\beta} - \frac{8g(0)}{f'(0)}\gamma^a{}_\alpha{}^\beta\epsilon^{bc}\gamma^5{}_\beta{}^\gamma D_a D_c\chi_{b\gamma}.\end{aligned}\tag{3.11}$$

The $\Lambda|$ equation is the analog of expression (2.19) in the pure bosonic case. There we saw that it is roughly R that is propagating. Here we see that it is a mixture of both R and A that make up the second propagating scalar field. The fermionic degree of freedom, $\Lambda|_\theta$, contains the first- as well as the second-derivative of the gravitino. These complicated expressions for the propagating degrees of freedom show how powerful the method of Legendre transformations really is.

As stated above, both Φ and Λ are propagating superfields. This can be made more explicit if we put the above action (3.8) in canonical form by performing a super-Weyl transformation to grow an explicit kinetic energy term for Λ . Consider the super-Weyl transformation discussed in the Appendix,

$$\begin{aligned}\bar{E} &= e^\Lambda E, \\ \bar{S} &= e^{-\Lambda} S + ie^{-\Lambda} \mathcal{D}^\alpha \mathcal{D}_\alpha \Lambda.\end{aligned}\tag{3.12}$$

Under such a transformation, action (3.8) becomes

$$S = 2i \int d^2x d^2\theta \bar{E} [e^\Lambda \bar{S} + ie^\Lambda \bar{\mathcal{D}}^\alpha \Lambda \bar{\mathcal{D}}_\alpha \Lambda + ig(\Phi) \bar{\mathcal{D}}^\alpha \Phi \bar{\mathcal{D}}_\alpha \Phi + e^{-\Lambda} f(\Phi) - \Phi].\tag{3.13}$$

This is the supersymmetric extension of the bosonic action (2.13).

In the following, we will drop the bar for notational simplicity. The Superfields Φ and Λ can be expanded into component fields as

$$\begin{aligned}\Phi &= \phi + i\theta^\alpha \pi_\alpha + \frac{i}{2} \bar{\theta} \theta F, \\ \Lambda &= \lambda + i\theta^\alpha \xi_\alpha + \frac{i}{2} \bar{\theta} \theta G.\end{aligned}\tag{3.14}$$

Inserting these expressions, as well as the expansions of E and S , into (3.13) gives a component field Lagrangian in which the fields A , F , and G are auxiliary. The equations of motion

of these fields are

$$\begin{aligned} A &= 2e^{-\lambda}\phi - e^{-2\lambda}f(\phi) + i\xi^\alpha\xi_\alpha, \\ F &= \frac{1}{4g(\phi)} \left[-1 + e^{-\lambda}f(\phi) - ig'(\phi)\pi^\alpha\pi_\alpha \right], \\ G &= \frac{1}{2}e^{-\lambda}\phi - \frac{1}{2}e^{-2\lambda}f(\phi) + \frac{i}{2}\xi^\alpha\xi_\alpha. \end{aligned} \quad (3.15)$$

Substituting these expressions for A , F , and G into the Lagrangian gives the component field Lagrangian for the propagating graviton $e_m{}^a$, gravitino $\chi_a{}^\alpha$, scalar fields ϕ and λ , and their fermionic partners π and ξ . We get a Lagrangian of the form

$$\mathcal{L} = \mathcal{L}_{\text{Boson}} + \mathcal{L}_{\text{Fermion}} + \mathcal{L}_{\text{Boson-Fermion}}. \quad (3.16)$$

The bosonic part of the Lagrangian is

$$\mathcal{L}_{\text{Boson}} = e \left[e^\lambda R - 2e^\lambda (\nabla\lambda)^2 - 2g(\phi)(\nabla\phi)^2 - V(\phi, \lambda) \right], \quad (3.17)$$

where the potential V is given by

$$V(\phi, \lambda) = \frac{1}{8g(\phi)} \left[1 - 2e^{-\lambda}(f'(\phi) + 2\phi^2 g(\phi)) + e^{-2\lambda}(f'(\phi)^2 + \phi f(\phi)g(\phi)) \right]. \quad (3.18)$$

The $\mathcal{L}_{\text{Fermion}}$ and $\mathcal{L}_{\text{Boson-Fermion}}$ parts of the Lagrangian are rather complicated and will not be presented here. Ignoring the fermionic fields, the bosonic equations of motion can be deduced from (3.17) and (3.18). They are given by

$$\begin{aligned} R &= -2(\nabla\lambda)^2 - 4\nabla^2\lambda + e^{-\lambda} \frac{\partial}{\partial\lambda} V(\phi, \lambda) \\ 2g'(\phi)(\nabla\phi)^2 + 4g(\phi)\nabla^2\phi &= \frac{\partial}{\partial\phi} V(\phi, \lambda) \\ \nabla_m \nabla_n e^\lambda - g_{mn} \nabla^2 e^\lambda - e^\lambda g_{mn} (\nabla\lambda)^2 + 2e^\lambda \nabla_m \lambda \nabla_n \lambda \\ -g(\phi)g_{mn}(\nabla\phi)^2 + 2g(\phi)\nabla_m \phi \nabla_n \phi &= \frac{1}{2}g_{mn} e^{-2\lambda} V(\phi, \lambda) \end{aligned} \quad (3.19)$$

for fields λ , ϕ and g_{mn} respectively. For constant fields ϕ_0 and λ_0 , these equations reduce to

$$\begin{aligned} \left. \frac{\partial V}{\partial\lambda} \right|_{(\phi_0, \lambda_0)} &= e^{\lambda_0} R \\ \left. \frac{\partial V}{\partial\phi} \right|_{(\phi_0, \lambda_0)} &= 0 \\ V(\phi_0, \lambda_0) &= 0 \end{aligned} \quad (3.20)$$

It follows that the equations specifying the constant vacua of the theory are $\partial V/\partial\phi|_{\phi_0} = 0$ and $V(\phi_0, \lambda_0) = 0$, whereas, in general, $\partial V/\partial\lambda|_{\lambda_0}$ is arbitrary. It is only if we demand that the vacuum has vanishing cosmological constant, that is $R = 0$, that $\partial V/\partial\lambda|_{\lambda_0} = 0$. We emphasize again that these conditions are different than the associated conditions for theories in higher-dimensional spacetimes. We will investigate these vacua in the next section.

4. NON-TRIVIAL VACUA AND SUPERSYMMETRY BREAKING

In this section, we would like to simplify the problem of finding the vacua of the higher-derivative supergravity by considering a concrete example for the functions f and g . Having made such a choice, we will evaluate the scalar potential $V(\phi, \lambda)$ and look for a constant vacuum state $\phi = \phi_0$ and $\lambda = \lambda_0$ with zero cosmological constant. At such a minimum, one has to make sure that g is non-negative to avoid having a ghost superfield Φ . The simple choice of $g(S)$ to be a positive constant suffices in this regard. Furthermore, one sees from (3.18) that linear or higher-order terms in g yield a rational potential V as a function in ϕ , making the problem less tractable. For these reasons, we will chose

$$g(S) = c, \quad (4.1)$$

where c is a real positive constant. For simplicity, we will chose f to be a general cubic polynomial in S with real coefficients

$$f(S) = a + S + bS^2 + dS^3. \quad (4.2)$$

The coefficient of the linear term S can be chosen to be unity by adjusting the overall normalization of the action.

With these choices, the potential energy (3.18) becomes

$$V = \frac{1}{8c} \left\{ 1 - 2 \left[1 + 2b\phi + (2c + 3d)\phi^2 \right] e^{-\lambda} + \left[1 + 4(b + ac)\phi + 2(2b^2 + 2c + 3d)\phi^2 + 4b(c + 3d)\phi^3 + d(4c + 9d)\phi^4 \right] e^{-2\lambda} \right\}. \quad (4.3)$$

We now solve generically for constant vacua of this theory with vanishing cosmological constant. It follows from (3.20) that we must solve the equations

$$\begin{aligned} \frac{\partial V}{\partial \lambda} &= 0, \\ \frac{\partial V}{\partial \phi} &= 0, \\ V(\phi, \lambda) &= 0. \end{aligned} \quad (4.4)$$

For any values of the parameters a, b, c, d there exists an extremum at $\phi_0 = \lambda_0 = 0$. The potential V vanishes at this point, without the need to adjust any of the parameters. We will refer to this point as the trivial extremum. We are interested to see if other, non-trivial, extrema with vanishing potential V exist. We find that there is precisely one non-trivial

extremum given by

$$\begin{aligned}\phi_0 &= \frac{-2b}{3(c+2d)}, \\ 1 - e^{-\lambda_0} &= \frac{-4b^2(c+3d)}{9c^2 - 4b^2(c+3d) + 36d(c+d)}, \\ a &= \frac{4}{27} \frac{b^3}{(c+2d)^2},\end{aligned}\tag{4.5}$$

for every choice of the parameters b, c , and d . The condition on a is required to ensure that the potential V is zero. We will, henceforth, restrict our discussion to the class of theories satisfying this condition on a . Any theory in this class is parametrized by b, c , and d . When $b \neq 0$, the theory has two distinct extrema. However, when $b = 0$ the two extrema become identical and the theory has only the trivial extremum at $\phi_0 = \lambda_0 = 0$. For different values of parameters b, c, d the non-trivial extremum (4.5) can be a local maximum, a saddle point, or a minimum. We will come back to this point later.

The supersymmetry transformation laws for the fermions, given in (A.15) and (A.16) in the Appendix, are

$$\begin{aligned}\delta\chi_{m\alpha} &= 2(\partial_m\tau_\alpha + \tfrac{1}{2}\omega_m\gamma^\beta_\alpha\tau_\beta) + \tfrac{1}{2}\gamma_{m\alpha}^\beta A\tau_\beta, \\ \delta\pi_\alpha &= [\gamma^m_\alpha{}^\beta(\partial_m\phi - \tfrac{i}{2}\chi_m^\gamma\pi_\gamma)]\tau_\beta - F\tau_\alpha, \\ \delta\xi_\alpha &= [\gamma^m_\alpha{}^\beta(\partial_m\lambda - \tfrac{i}{2}\chi_m^\gamma\xi_\gamma)]\tau_\beta - G\tau_\alpha,\end{aligned}\tag{4.6}$$

where τ_α is the supersymmetry transformation parameter. Note that if any of the auxiliary fields A, F, G develops a non-vanishing vacuum expectation value, the corresponding fermion will develop an inhomogeneous piece in its supersymmetry transformation law, which signals supersymmetry breaking. Using (3.15), (4.1), and (4.2), we find that the auxiliary fields all vanish at the trivial extremum $\phi_0 = \lambda_0 = 0$. Hence, supersymmetry is never broken at that point. On the other hand, the auxiliary fields evaluated at the extremum (4.5) are given by

$$\begin{aligned}A_0 &= -\frac{18b(24d^3 + 36d^2c - 8b^2d^2 - 4b^2dc + 18dc^2 + 3c^3)}{P^2}, \\ F_0 &= -\frac{2b^2}{P}, \\ G_0 &= -\frac{12b^3c(c+2d)}{P^2},\end{aligned}\tag{4.7}$$

where P is a polynomial in b, c , and d given by

$$P = c(9c - 4b^2 + 36d) - 12d(b^2 - 3d).\tag{4.8}$$

Note that apart from the case $b = 0$, which makes the extremum (4.5) coincide with trivial extremum $\phi_0 = \lambda_0 = 0$, at least one of these vacuum expectation values of the auxiliary

fields, namely F_0 , is non-zero. We will assume from now on that $b \neq 0$. With such a choice, we can conclude that supersymmetry is broken at the non-trivial extremum (4.5).

Since supersymmetry is broken, there should exist a massless fermion in the theory, a goldstino. In order to see this, we need to consider the fermion mass-matrix. The fermionic part of the Lagrangian, $\mathcal{L}_{\text{Fermion}}$, splits into three terms, the kinetic energy term, the mass term, which is quadratic in the fermions, and a four-fermi interaction term. The fermionic mass term, evaluated at the non-trivial extremum (4.5), is given by

$$\begin{aligned} \mathcal{L}_{\text{M-Fermion}} = e \big(& m_{11} i \pi^\alpha \pi_\alpha + m_{22} i \zeta^\alpha \zeta_\alpha + m_{33} \epsilon^{ab} \chi_a^\alpha \gamma_{\alpha}^5{}^\beta \chi_{b\beta} + 2m_{12} i \pi^\alpha \zeta_\alpha \\ & + 2m_{13} i \pi^\alpha \gamma_{\alpha}^a{}^\beta \chi_{a\beta} + 2m_{23} i \zeta^\alpha \gamma_{\alpha}^a{}^\beta \chi_{a\beta} \big), \end{aligned} \quad (4.9)$$

where

$$\begin{aligned} m_{11} &= \frac{9bc(c+2d)}{P}, \\ m_{22} &= \frac{-2b(-2b^2+3c+6d)}{P}, \\ m_{33} &= \frac{2b^3c}{3(c+2d)P}, \\ m_{12} &= \frac{-3c(3c-4b^2+12d)+12d(b^2-3d)}{2P}, \\ m_{13} &= \frac{-2b^2c}{P}, \\ m_{23} &= \frac{bc(9c-8b^2+36d)-12bd(b^2-3d)}{6(c+2d)P}, \end{aligned} \quad (4.10)$$

and P is the polynomial defined in (4.8). We can diagonalize the fermion mass matrix as follows. First define

$$\tilde{\chi}_{a\alpha} = \chi_{a\alpha} + 2\gamma_{a\alpha}{}^\beta (m_{13}\pi_\beta + m_{23}\xi_\beta). \quad (4.11)$$

Then

$$\begin{aligned} m_{33} i \epsilon^{ab} \tilde{\chi}_a^\alpha \gamma_{\alpha}^5{}^\beta \tilde{\chi}_{b\beta} = & m_{33} i \epsilon^{ab} \chi_a^\alpha \gamma_{\alpha}^5{}^\beta \chi_{b\beta} + 2m_{13} i \pi^\alpha \gamma_{\alpha}^a{}^\beta \chi_{a\beta} + 2m_{23} i \zeta^\alpha \gamma_{\alpha}^a{}^\beta \chi_{a\beta} \\ & + 2 \frac{m_{13}^2}{m_{33}} i \pi^\alpha \pi_\alpha + 2 \frac{m_{23}^2}{m_{33}} i \zeta^\alpha \zeta_\alpha + 4 \frac{m_{13}m_{23}}{m_{33}} i \pi^\alpha \xi_\alpha. \end{aligned} \quad (4.12)$$

Note that we have used the assumption that $b \neq 0$, which implies supersymmetry is broken, since otherwise m_{33} would be zero and the above computation would break down. However, keeping this assumption in mind, we can proceed and substitute (4.12) into (4.9). The fermion mass term, then, takes the form

$$\mathcal{L}_{\text{M-Fermion}} = e \big(m_{33} i \epsilon^{ab} \tilde{\chi}_a^\alpha \gamma_{\alpha}^5{}^\beta \tilde{\chi}_{b\beta} + \tilde{m}_{11} i \pi^\alpha \pi_\alpha + \tilde{m}_{22} i \xi^\alpha \xi_\alpha + 2\tilde{m}_{12} i \pi^\alpha \xi_\alpha \big), \quad (4.13)$$

where

$$\begin{aligned}\tilde{m}_{11} &= m_{11} - 2\frac{m_{13}^2}{m_{33}}, \\ \tilde{m}_{22} &= m_{22} - 2\frac{m_{23}^2}{m_{33}}, \\ \tilde{m}_{12} &= m_{12} - 4\frac{m_{13}m_{23}}{m_{33}}.\end{aligned}\tag{4.14}$$

Since the “shifted” gravitino $\tilde{\chi}_a{}^\alpha$ is a mixture of the original fields $\chi_\mu{}^\alpha$, π^α , and ξ^α , its supersymmetry transformation law changes accordingly. The auxiliary field part of its transformation, evaluated at the non-trivial extremum (4.5), is now given by

$$\delta\tilde{\chi}_{a\alpha} = \gamma_{a\alpha}{}^\beta \left(\frac{1}{2}A_0 + m_{13}F_0 + m_{23}G_0\right) \tau_\beta = 0.\tag{4.15}$$

In other words, we find that the shifted gravitino transforms homogeneously. We now diagonalize the 2×2 mass matrix for the fermions π_α and ξ_α . We find that it has the two eigenfields

$$\begin{aligned}\tilde{\pi}_\alpha &= \pi_\alpha + h\xi_\alpha, \\ \tilde{\xi}_\alpha &= \xi_\alpha - h\pi_\alpha,\end{aligned}\tag{4.16}$$

where h is given by

$$h = -\frac{P}{6bc(c+2d)}.\tag{4.17}$$

The fermion $\tilde{\pi}$ has a non-vanishing mass given by \tilde{m}_{11} , whereas the mass of fermion $\tilde{\xi}$ vanishes. The auxiliary-field piece in the supersymmetry transformation law of the new field $\tilde{\pi}$ is given by

$$\delta\tilde{\pi}_\alpha = -(F_0 + hG_0)\tau_\alpha = 0,\tag{4.18}$$

which means that $\tilde{\pi}$ transforms homogeneously. On the other hand, $\tilde{\xi}$ transforms with an auxiliary field piece given by

$$\begin{aligned}\delta\tilde{\xi}_\alpha &= -(G_0 - hF_0)\tau_\alpha \\ &= -(1 + h^2)G_0\tau_\alpha \neq 0.\end{aligned}\tag{4.19}$$

That is, $\tilde{\xi}$ transforms inhomogeneously. The diagonalization of the fermion mass term is now complete. The diagonal form can be written as

$$\mathcal{L}_{\text{M-Fermion}} = e \left(m_{33} i \epsilon^{ab} \tilde{\chi}_a{}^\alpha \gamma_\alpha{}^5{}^\beta \tilde{\chi}_{b\beta} + \tilde{m}_{11} i \tilde{\pi}^\alpha \tilde{\pi}_\alpha \right).\tag{4.20}$$

The vanishing mass for $\tilde{\xi}$ implies that $\tilde{\xi}$ is a Goldstone fermion, in accordance with the spontaneously broken supersymmetry at this extremum. This conclusion is further strengthened by the fact that the gravitino, $\tilde{\chi}_a{}^\alpha$, has acquired a non-vanishing mass. The only field which transform inhomogeneously is the massless Goldstone fermion $\tilde{\xi}^\alpha$.

The above analysis is true at the non-trivial extremum (4.5), no matter whether it is a local maximum, saddle point, or minimum. However, we are specifically interested in the stable vacuum of the theory. The fact that any non-trivial theory of gravitation in two dimensions contains a ghost-like degree of freedom, as discussed in Section 2, makes it unclear as to exactly what one means by stable vacuum state. However, for the theory under investigation, if we turn gravity off, we still have two physical degrees of freedom. In this case, we can require that the theory be stable at an extremum of the potential in the usual sense; that is, any fluctuation of the fields around the extremum should increase the energy. This is true if and only if the extremum locally minimizes the potential. We find that the non-trivial extremum (4.5) is a local minimum of the potential when the two conditions

$$\begin{aligned} 2d + c &< 0, \\ 48d^3 + 68d^2c + 32dc^2 + 5c^3 &< 0, \end{aligned} \tag{4.21}$$

are simultaneously satisfied. For example, the choice $b = c = -d = 1$, which satisfy the above two conditions, makes the 2×2 Hessian scalar mass matrix positive definite, and, hence, ensures that (4.5) is a local minimum. It follows that there exists a class of theories in which there is a non-trivial stable vacuum state with zero cosmological constant which breaks the $(1, 1)$ supersymmetry spontaneously. This result will obviously continue to hold for more general choices of the functions $f(S)$ and $g(S)$.

In a previous paper, we considered the most general theory of quadratic bosonic gravitation in four dimensions [1]. We showed that the only stable vacuum in this theory is the trivial vacuum with vanishing cosmological constant. In a subsequent paper, we generalized our results to higher-derivative bosonic gravitation beyond the quadratic level [2]. We showed that such theories still possess a trivial vacuum with vanishing cosmological constant but, unlike the quadratic case, they generically have non-trivial vacua as well. These vacua, however, are always characterized by a non-vanishing cosmological constant. Hence, any non-trivial vacua in these bosonic theories must be deSitter or anti-deSitter spaces, typically with a radius of curvature of the order of the inverse Planck mass. For this reason, such theories are of little interest to particle physics. We find that exactly analogous behavior occurs in the bosonic two-dimensional higher-derivative gravity theories discussed in Section 2 of this paper. Let us digress briefly to make these bosonic two-dimensional properties explicit, before returning to the supergravitational case. It turns out to be more convenient to discuss these properties using the original metric g_{mn} rather than the conformally transformed metric \bar{g}_{mn} . Therefore, we will use metric g_{mn} for the remainder of this section. Consider the quadratic bosonic action (2.26). We reduce this theory to second-order form by introducing a single scalar field λ with the potential energy given by (2.31). Combining (2.28) and (2.29), we

find that the relation between λ and R is

$$e^\lambda = 1 + 2\epsilon R. \quad (4.22)$$

If we demand that spacetime has vanishing cosmological constant, or, what is the same thing, $R = 0$, then we must have $\lambda_0 = 0$. In order for this to be a vacuum of the theory, it follows from (2.5) that the potential energy must satisfy $V(0) = 0$. Using the potential (2.31), one can easily verify that this is the case. Moreover, $\lambda_0 = 0$ is the only zero of potential (2.31). Hence, there are no other vacua, whether they correspond to flat spacetime or not. This is exactly equivalent to the bosonic four-dimensional quadratic gravity case. Now consider the more general higher-derivative bosonic gravitation of action (2.15). This theory, once again, is reduced to second-order form by the introduction of a single scalar field λ . The relation between R and λ is now given by the more complicated equation (2.19), namely

$$e^\lambda = f'(R). \quad (4.23)$$

Although this relation is more general than the quadratic case (4.22), the fact remains that the zero cosmological constant condition, $R = 0$, corresponds to a unique solution at $\lambda_0 = \ln f'(0)$. This can, without loss of generality, always be normalized to $\lambda_0 = 0$. As before, it follows from (2.5) that this point is a vacuum of the theory only if the potential energy satisfies $V(0) = 0$, which is equivalent to the condition $f(0) = 0$. Thus, provided there is no constant piece in the function f , we find that the theory has a trivial vacuum state at $\lambda_0 = 0$. Generally, the potential function is considerably more complicated than the simple potential (2.31) in the quadratic case. Unlike the quadratic potential (2.31), it generically has more than one zero. However, from the condition (4.23), we see that any non-trivial vacuum different from $\lambda_0 = 0$ will not correspond to flat spacetime. Instead, it will have a constant non-vanishing curvature R and correspond to deSitter or anti-deSitter spacetime with non-zero cosmological constant. Again, this is in direct analogy with the associated four-dimensional case.

Given these results for bosonic gravitation, it appears all the more remarkable that, in this paper, we have shown that two-dimensional higher-derivative supergravitational theories allow non-trivial vacua with vanishing cosmological constant. In fact, as we have discussed, even quadratic supergravity theories possess non-trivial vacua corresponding to $R = 0$. It follows that such theories are far more relevant to particle physics. It is worth, therefore, a discussion of why supersymmetric theories differ from bosonic ones in this crucial issue.

Let us consider the case of quadratic supersymmetric gravitation given by (3.5). The reduction of this theory to second-order form (3.8) requires the introduction of two supermultiplets with scalar degrees of freedom ϕ and λ . In order to deduce the relation between ϕ , λ , and R , we expand (3.8) into component form and compute the equations of motion for

F , G , and λ . Eliminating the auxiliary fields from the λ equation gives the desired relation

$$R = \frac{1}{4g(\phi)} \left(e^\lambda - f'(\phi) - \frac{1}{2}\phi^2 g(\phi) \right). \quad (4.24)$$

This relation is the supersymmetric analog of (4.22). It should be clear that requiring R to vanish no longer singles out a unique choice of ϕ and λ . Indeed, there may be many values of ϕ and λ that are compatible with $R = 0$. In the particular case we studied, where g and f are given by (4.1) and (4.2) respectively, the above relation becomes

$$R = \frac{1}{4c} \left(e^\lambda - 1 - 2b\phi - (3d + 2c)\phi^2 \right). \quad (4.25)$$

It can be easily verified that both the trivial fields $\phi_0 = \lambda_0 = 0$ and the non-trivial fields (4.5) are compatible with $R = 0$. Furthermore, both the trivial and non-trivial solutions satisfy, by construction, the conditions $V(\phi_0, \lambda_0) = 0$ and $\partial V / \partial \phi|_{(\phi_0, \lambda_0)} = 0$ and, hence, are vacua of the theory. It is clear that it is the new degree of freedom ϕ , which is introduced by supersymmetry, that allows non-trivial vacuum solutions corresponding to zero cosmological constant to exist. As shown in (3.10), ϕ is directly related to the “auxiliary” field A of the $(1, 1)$ supergravity multiplet that propagates, and becomes physical, in higher-derivative theories. The possibility of non-trivial vacuum solutions with zero cosmological constant makes higher-derivative supergravitation much more relevant to particle physics than the bosonic theories previously discussed.

5. CONCLUSIONS

We have constructed the most general quadratic $(1, 1)$ supergravitation theory in two dimensions. We have shown that this theory is reducible to a second-order form by the introduction of two real scalar supermultiplets. We have evaluated the scalar potential for the second-order theory and presented an explicit class of examples which possess a non-trivial stable vacuum state with zero cosmological constant that spontaneously breaks the $(1, 1)$ supersymmetry. This result is quite general and leads to the main conclusion of this paper: two-dimensional $(1, 1)$ supergravity theories generically possess stable, flat spacetime, but non-trivial, vacua that spontaneously break supersymmetry. Supersymmetry breakdown is due to non-trivial vacuum expectation values for the extra scalar degrees of freedom that arise directly from the super-zweibein in higher-derivative theories. That is, supersymmetry is broken by supergravity itself.

In our opinion, this result represents a new approach to the theory of spontaneous supersymmetry breaking. If it could be extended to four-dimensional $N = 1$ supergravity, this new method of supersymmetry breaking would have obvious applications to phenomenological supersymmetric theories. With this in mind, we have recently shown that, indeed, exactly the same phenomenon occurs in $D = 4$, $N = 1$ quadratic supergravitation [5]. It follows that

higher-derivative supergravity might serve as a natural mechanism for spontaneously breaking supersymmetry in phenomenologically interesting particle physics models. The results of our ongoing investigations will be presented elsewhere.

APPENDIX: TWO-DIMENSIONAL (1,1) SUPERSPACE

The structure of (1,1) superspace was studied by Howe [7]. The (1,1) superspace has coordinates $z^M = (x^m, \theta^\mu)$, where m and μ can both take on two values. We will use (m, n, \dots) for spacetime indices, (a, b, \dots) for tangent space indices, and (α, β, \dots) for spinor indices. The bosonic metric and the anti-symmetric tensor are

$$\begin{aligned} \eta_{ab} &= \text{diag}(-1, +1), & \epsilon_{ab} &= -\epsilon_{ba}, & \epsilon_{01} &= 1, \\ \epsilon^{ab} &= -\epsilon_{ab}, & \epsilon_{ab}\epsilon^{bc} &= \delta_a^c. \end{aligned} \quad (\text{A.1})$$

The fermionic anti-symmetric “metric” is given by

$$\begin{aligned} \epsilon_{\alpha\beta} &= \epsilon^{\alpha\beta}, & \epsilon_{12} &= 1 = -\epsilon_{21}, & \epsilon_{11} &= \epsilon_{22} = 0, \\ \epsilon_{\alpha\beta}\epsilon^{\beta\gamma} &= -\delta_\alpha^\gamma, & \epsilon_{\alpha\beta}\epsilon^{\alpha\beta} &= 2. \end{aligned} \quad (\text{A.2})$$

The γ -matrices are chosen to be real, satisfying

$$\gamma^a{}_\alpha{}^\beta \gamma^b{}_\beta{}^\gamma = \eta^{ab} \delta_\alpha^\gamma - \epsilon^{ab} \gamma^5{}_\alpha{}^\gamma. \quad (\text{A.3})$$

where $\gamma^5 = \gamma^0 \gamma^1$. This implies the relations

$$[\gamma^a, \gamma^b] = -2\epsilon^{ab} \gamma^5, \quad [\gamma^a, \gamma^5] = 2\epsilon^{ab} \gamma_b, \quad \gamma^a \gamma^5 = \epsilon^{ab} \gamma_b. \quad (\text{A.4})$$

An explicit representation is given by

$$\gamma^0{}_\alpha{}^\beta = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \gamma^1{}_\alpha{}^\beta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^5{}_\alpha{}^\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (\text{A.5})$$

The geometry of the (1,1) superspace is determined by the super-zweibein E_M^A and the connection Ω_B^A . There are two important two-forms, the torsion and curvature defined by

$$\begin{aligned} T^A &= \mathcal{D}E^A = \frac{1}{2} E^C \wedge E^B T_{BC}^A, \\ R_A^B &= d\Omega_A^B + \Omega_A^C \wedge \Omega_C^B = \frac{1}{2} E^D \wedge E^C R_{CD,A}^B. \end{aligned} \quad (\text{A.6})$$

They satisfy the Bianchi identities,

$$\begin{aligned} \mathcal{D}T^A &= E^B \wedge R_B^A, \\ \mathcal{D}R_A^B &= 0. \end{aligned} \quad (\text{A.7})$$

Howe imposed the following set of constraint on the supertorsion

$$T_{\beta\gamma}{}^a = 2i\gamma^a{}_{\beta\gamma}, \quad T_{\beta\gamma}{}^\alpha = T_{bc}{}^a = 0. \quad (\text{A.8})$$

He found that all the components of the torsion and curvature can then be written in terms of a single superfield S . If S vanishes, so does the curvature and the space is flat. In

Wess-Zumino gauge, every tensor can be expressed in terms of only three component fields, namely, the zweibein e_m^a , the Rarita-Schwinger field χ_m^α , and an “auxiliary” scalar field A . The supervolume element E is given by

$$E = e \left(1 + \frac{i}{2} \theta^\alpha \gamma^\alpha{}_\alpha{}^\beta \chi_{a\beta} + \bar{\theta}\theta \left[\frac{i}{4} A + \frac{1}{8} \epsilon^{ab} \chi_a{}^\alpha \gamma^\alpha{}_\alpha{}^\beta \chi_{b\beta} \right] \right), \quad (\text{A.9})$$

where $\bar{\theta}\theta = \theta^\alpha \theta_\alpha$. The superfield S is given by

$$S = A + i\theta^\alpha \Sigma_\alpha + \frac{i}{2} \bar{\theta}\theta C, \quad (\text{A.10})$$

where

$$\begin{aligned} C &= -R - \frac{1}{2} \chi_a{}^\alpha \gamma^\alpha{}_\alpha{}^\beta \psi_\beta + \frac{i}{4} \epsilon^{ab} \chi_a{}^\alpha \gamma^\alpha{}_\alpha{}^\beta \chi_{b\beta} A - \frac{1}{2} A^2, \\ \Sigma_\alpha &= -2\epsilon^{ab} \gamma^5{}_\alpha{}^\beta \mathcal{D}_a \chi_{b\beta} - \frac{1}{2} \gamma^a{}_\alpha{}^\beta \chi_{a\beta} A. \end{aligned} \quad (\text{A.11})$$

Howe showed that the generalization of Weyl transformations to superspace, compatible with the above constraints, is given by the super-Weyl transformations

$$\begin{aligned} \bar{E}_M^a &= \Lambda E_M^a, \\ \bar{E}_M{}^\alpha &= \Lambda^{1/2} E_M{}^\alpha - \frac{i}{2} \Lambda^{-1/2} E_M^a \gamma_a{}^{\alpha\beta} \mathcal{D}_\beta \Lambda, \\ \bar{E}_a{}^M &= \Lambda^{-1} E_a{}^M + i\Lambda^{-2} \gamma_a{}^{\alpha\beta} \mathcal{D}_\beta \Lambda E_\alpha{}^M, \\ \bar{E}_\alpha{}^M &= \Lambda^{-1/2} E_\alpha{}^M. \end{aligned} \quad (\text{A.12})$$

One can compute the change in the superfield S under these transformations, finding

$$\bar{S} = \Lambda^{-1} S + i\Lambda^{-3} \mathcal{D}_\alpha \Lambda \mathcal{D}^\alpha \Lambda - i\Lambda^{-2} \mathcal{D}_\alpha \mathcal{D}^\alpha \Lambda. \quad (\text{A.13})$$

Howe also showed that every $(1, 1)$ superspace is super-conformally flat.

We will consider theories of supergravity coupled to matter. Matter superfields Φ are real scalar superfields having an expansion of the form

$$\Phi = \phi + i\theta^\alpha \pi_\alpha + \frac{i}{2} \bar{\theta}\theta F. \quad (\text{A.14})$$

For these fields we have a choice for their super-Weyl weight. We will choose zero super-Weyl weight for all the matter fields considered in this paper.

Howe also deduced the supersymmetry transformations for the gravitational multiplet,

$$\begin{aligned} \delta e_m^a &= i\tau^\alpha \gamma^\alpha{}_\alpha{}^\beta \chi_{m\beta}, \\ \delta \chi_{m\alpha} &= 2(\partial_m \tau_\alpha + \frac{1}{2} \omega_m \gamma^\alpha{}_\alpha{}^\beta \tau_\beta) + \frac{1}{2} \gamma_{m\alpha}{}^\beta A \tau_\beta, \\ \delta A &= i\tau^\alpha \psi_\alpha. \end{aligned} \quad (\text{A.15})$$

where τ_α is the gauge parameter and ω_m is the spin connection. We will also need the supersymmetry transformations of a matter superfield Φ , which are given by

$$\begin{aligned}\delta\phi &= i\tau^\alpha\pi_\alpha, \\ \delta\pi_\alpha &= [\gamma^m{}_\alpha{}^\beta(\partial_m\phi - \tfrac{i}{2}\chi_m{}^\gamma\pi_\gamma)]\tau_\beta - F\tau_\alpha, \\ \delta F &= i\tau^\alpha\gamma^m{}_\alpha{}^\beta [-(\partial_m\pi_\beta + \tfrac{1}{2}\omega_m\gamma^5{}_\beta{}^\gamma\pi_\gamma) + \tfrac{1}{2}\gamma^n{}_\beta{}^\gamma(\partial_n\phi - \tfrac{i}{2}\chi_n{}^\delta\pi_\delta)\chi_{m\gamma} - \tfrac{1}{2}F\chi_{m\beta}].\end{aligned}\tag{A.16}$$

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